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BUCKLING OF SHELLS WITH CUTOUTS, EXPERIMENT AND ANALYSIS†

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Abstract—Axial compression tests were performed on eleven thin-walled aluminum cylinders with rectangular cutouts. Various types of cutout reinforcement were installed on seven of the test specimens. The test results are compared with the cylinder buckling loads prior to installation of the cutouts, and correlated with computer-predicted failure loads. The latter were based on the use of the STAGS computer program.

For thin cylinders such as these, the test and computer-based analysis shows that for small to moderate size cutouts, reinforcement of the cutout is of no benefit unless the cylinder is of extremely high (geometrical) quality. For cylinder quality and cutout size where reinforcement is beneficial, the relative merits of the various reinforcement configurations are discussed and an empirical basis for design is proposed.

INTRODUCTION

THE calculation of collapse loads for shells with cutouts requires a nonlinear analysis and has until very recently been beyond the state of the art in shell analysis. The large number of parameters makes it impossible to produce design charts by use of a purely empirical approach and a theoretical analysis has been prohibited by excessive computer costs. Consequently, design of cutout reinforcements has been based on rules of thumb which are generally quite conservative due to the uncertainty involved. However, recent improvements in computer technology as well as in numerical analysis methods have brought the computer cost down to a level where it now appears feasible to establish design procedures with a more solid foundation.

The first nonlinear analysis of cylindrical shells with rectangular cutouts was presented in Ref. [1]. That work was based on a computer code, STAGS. At that time it was not economically feasible to analyze shells which were thin enough for collapse to occur in the elastic range. This essentially made meaningful comparisons impossible between test and theory; at least for metal cylinders. Later improvements (Refs. [2, 3]) have not only extended the generality of the STAGS computer program but also improved its efficiency so that now it is possible to shed more light on the problem of the collapse of shells with cutouts through a combination of analytical and experimental investigations. The present paper is based on the results of experiments on aluminum cylinders and application of STAGS for both pre- and post-test analysis.

Eleven thin-walled aluminum cylinders with cutouts were tested in axial compression. In view of the sensitivity of axially loaded cylinders to small initial imperfections, it was

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necessary to test each cylinder first without cutouts. This establishes a reference level for the cylinder which is needed for a proper understanding of the test result for the cylinder with cutouts. Damage to the specimen during these preliminary tests was avoided by use of a buckle limiting device, consisting of an electrically isolated mandrel mounted inside the cylinder. If the gap between the cylinder and mandrel is small enough, stresses in the buckled specimen will remain in the elastic range.

TEST SPECIMENS AND PROCEDURES

Specimen material and geometry

The eleven cylinders tested were machined from 6061-T6 aluminum tube stock. This extruded tubing raw stock has an outer diameter of 12·75 in. and an inner diameter of 11·75 in.

All cylinders were machined to the dimensions shown in Fig. 1, the thickness of the thin-walled portion being the only variable within the set of eleven specimens. The purpose of the end rings is to help distribute the load uniformly and to serve as an attachment ring for the buckle capture device. The figure also shows the cutout dimensions and one of the different reinforcement arrangements used in the program.

The test cylinders were measured for thickness variation at 24° stations around the circumference and at 1·75 in. intervals longitudinally, starting 1 in. from one of the end rings. A summary of thickness measurements is given in Tables 1 and 2. These tables list the

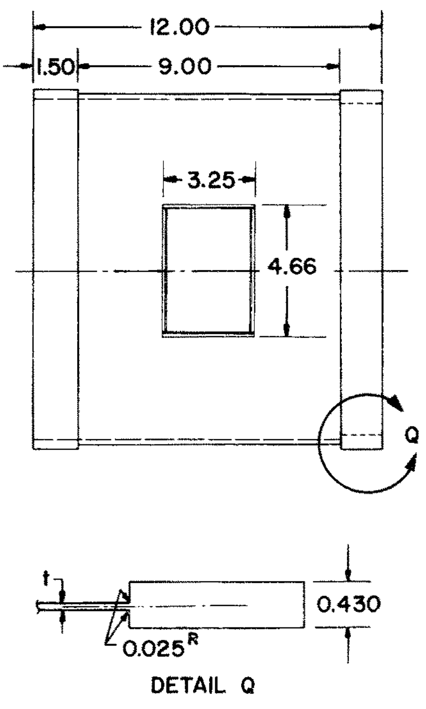


FIG. 1. Test specimen.

TABLE 1. 0·014-IN. THICK CYLINDERS

Cylinder No.	1	2	3	4	5	6	7
Thickness range (mils)	14/16	14/15	12/14	12/16	12/14	12/15	13/15
Average thickness (mils)	14·76	14·68	12·81	14·64	13·27	13·67	13·73
Classical buckling load† P_{CL} (lb)	7389	7389	7389	7389	7389	7389	7389
First buckling load P_0 (lb)	4450	4620	4500	3920	4180	4110	3075
P_0 as % of P_{CL}	60	63	61	53	57	56	42
"Repeatable" buckling load P_R (lb)	4030	4585	4280	3735	3970	3360	3055
Range of P_R (lb)	±70	±35	±170	±45	±150	±70	±15
P_R as % of P_{CL}	55	62	58	50	54	46	41
Arc of cutout (deg.)	30	45	45	45	45	45	45
Reinforcement type	None	None	None	*A*	*A*	*B*	*P*
Buckling load with cutout (lb)	2740	2540	2050	3190	2850	2560	2600

† $P_{CL} = 0{\cdot}6E(t/R) \,.\, 2\pi Rt$.

TABLE 2. 0·009-IN. THICK CYLINDERS

Cylinder No.	8	9	10	11
Thickness range (mils)	9/11	8/11	9/11	9/11
Average thickness (mils)	9·72	9·50	9·53	9·53
Classical buckling load† P_{CL} (lb)	3054	3054	3054	3054
First buckling load P_0 (lb)	1340	1480	1390	1590
P_0 as % of P_{CL}	44	49	46	52
"Repeatable" buckling load P_R (lb)	1265	1435	1375	1555
Range of P_R (lb)	±35	±20	±15	±35
P_R as % of P_{CL}	41	47	45	51
Arc of cutout (deg.)	45	45	45	45
Reinforcement type	None	*B*	*B*‡	*C*
Buckling load with cutout (lb)	807	1275	1030	1055

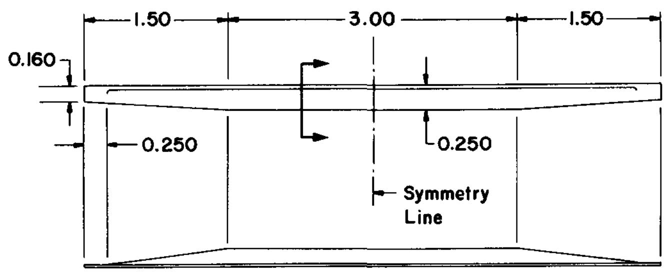
† $P_{CL} = 0{\cdot}6E(t/R)2\pi Rt$.
‡ Reinforcement on inside of cylinder.

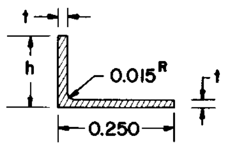
minimum and maximum thickness measured, and the average thickness, based on the seventy-five thickness measurements. Details of the thickness measurements are given in Ref. [4] which also gives information about the procedures for manufacture of the cylinders.

Following the preliminary tests, two rectangular cutouts were made on the cylinder. In each case, these were centered at the cylinder midheight and 180° apart on the circumference.

The cutouts were made by drilling 0·062 in. diameter holes at each corner of the proposed cutout, and then sawing along prescribed lines with a high-speed dental wheel. The wheel is driven by a hand-held Dremel motor. The cylinder is held in a felt-lined wood cradle and the operator's hand is braced on a bar fastened to the cradle. Some cleaning up and deburring with a swiss file is necessary. Because of the high speed of the abrasive wheel, almost no tool pressure is required. The size of the cutouts on all cylinders was 45° of arc by 3 in. in the axial direction except cylinder No. 1 which had cutouts with a 30° arc.

All reinforcement of the cutouts consisted of angle sections. Figure 2 shows the three basic types of reinforcement used. These very thin angles were machined from bar stock. A "back-up" bar is needed when machining the last outstanding leg. Thickness tolerance





Type	t	h
A	0.020	0.150
B	0.010	0.150
C	0.010	0.080
P*	0.020	0.150

*See Fig. 3

FIG. 2. Stringer reinforcement.

was $\pm 0{\cdot}001$ in. The figure also shows the tapered end details used in all reinforcements except that of cylinder No. 7. All reinforcement was installed on the outside surface of the cylinder with the exception of cylinder No. 10.

Figure 1 shows how angle reinforcement (with the same cross section as type A) was arranged as the "picture frame" around the cutout of cylinder No. 7. This is referred to as type P reinforcement.

In all tests, with or without cutouts, the cylinders were loaded by a screw-driven "SR-4, FGT" universal testing machine of 50,000 lb capacity. This machine has several loading ranges. The two ranges used were 2500 or 10,000 lb full scale. The resolution of this machine is 0·2 per cent of the "full scale" being used, and the error is less than either 0·5 per cent of the load reading or the resolution figure, whichever is larger.

The load is applied to the cylinder through a 2 in. thick aluminum end plate at each end of the cylinder. These square plates have their contacting surfaces machined to a flatness better than $\pm 0{\cdot}0005$ in.

One of the end loading plates (the lower one) is placed on top of the cross-head, the cylinder is placed over this and the other end loading plate at the top of this stack, then the upper plate is pulled down with a rod which passes through both loading plates, the cylinder and the cross-head and is connected to the platen of the test machine. The latter is then driven downwards to load the cylinder. In addition to the rod's flexibility, a two-axis flexure is added to this tension train, providing assurance that the upper loading plate is completely free to rotate about any axis. With close tolerances on the rod and through-holes, concentricity of the loading axis with the cylinder axis is also easier to insure.

The loading rate, which is not critical in tests such as these, was approximately 400 lb/min. All test cylinders were instrumented with strain gages in order to make possible detailed comparisons between test and theory. The loading was stopped at regular load intervals to permit scanning of these strain gages. During these stops, no unloading (or stress relaxation) was observed.

Test results

Tables 1 and 2 summarize some important test parameters together with test results for 0·014 and 0·009 in. thick cylinders, respectively. In these tables the classical buckling load is based on the nominal thickness.

In the preliminary tests on cylinders without cutouts, the cylinder with the buckle capturing mandrel in place was tested repeatedly. It was generally found that the initial test caused some damage to the cylinder, but that subsequent tests did not add to this damage. Consequently, for the second and following tests the buckling load was more or less repeatable. It is felt that this repeatable buckling load is the proper reference level to use when the effect of a cutout is evaluated.

Some strain measurements were performed on all tested cylinders. Cylinder No. 2 in particular was instrumented with a large number of strain gages. Some of the strain gage readings are shown here in the section on correlation. Complete tabulations of the strain gage readings are available in Ref. [4].

THEORETICAL RESULTS

Computer analysis was used in connection with this program for two different purposes. Pretest analysis is needed in order that the test specimen will be proportioned to give as much information as possible. Post-test analysis is needed for the enhancement of the understanding of the results obtained from the experiments. To a large degree the same computer runs could be used for both of these purposes and thus separate discussion of pre- and post-test analysis will not be undertaken. The theoretical results will be presented here and their influence on the choice of cutout geometry will be discussed. Correlation of experimental and theoretical results and a discussion of their significance will be presented below.

The computer program used in the analysis is STAGS, a program for the nonlinear analysis of shells of general shape. STAGS is based on an energy principle in combination with finite difference approximations. A detailed description of the program is given in Ref. [5].

For a thinner cylinder the finite difference grid must be finer and thus the computer time goes up. It appears that the price of the analysis is approximately inversely proportional to the square of the thickness. It is desirable then that the cylinders used in the program be as thick as possible, short of causing problems with inelastic deformations.

The first attempt at analysis was made for a shell with

$$R = 6{\cdot}06$$

$$t = 0{\cdot}020$$

$$\text{Cutout: } 30° \times 3 \text{ in.}$$

It was found that for such a cylinder, stresses around the cutout would reach the proportionality limit of the material at about half the elastic collapse load. A second attempt was therefore made with a thinner-walled cylinder, i.e. $t = 0{\cdot}014$. The critical load for this cylinder with a 30° cutout was 2650 lb/in. and examination of the stresses indicated that collapse would occur in the elastic range. However, the difference between the buckling load for a cylinder without cutout and one with unreinforced cutout was too narrow to permit a successful study of the efficiency of cutout reinforcements. Therefore, cylinders with 0·014 in. thickness and wider cutouts were also analyzed. The critical load for a 45° cutout was found to be 2250 lb and with a 60° cutout it was 1900 lb. The lateral displacements at the edge of the cutout for these three shells are shown as a function of applied load in Fig. 3. Although the results of Ref. [1] provided some guidance, two attempts had to be made before a suitable finite difference grid was established. The grid which finally was chosen is shown in Fig. 4. As no shells can be thicker than 0·014 in., tests were made on cylinders with the nominal thicknesses $t = 0{\cdot}014$ ($R/t = 430$) and $t = 0{\cdot}009$ ($R/t = 675$). Thicker shells would collapse in the inelastic range.

The first attempt at analysis of shells with reinforced cutouts was made with a shell thickness of 0·014 in. and a 60° by 3 in. cutout. The reinforcement chosen for this first analysis was of the same type as was used in the analysis of Ref. [1]. A solid rectangular

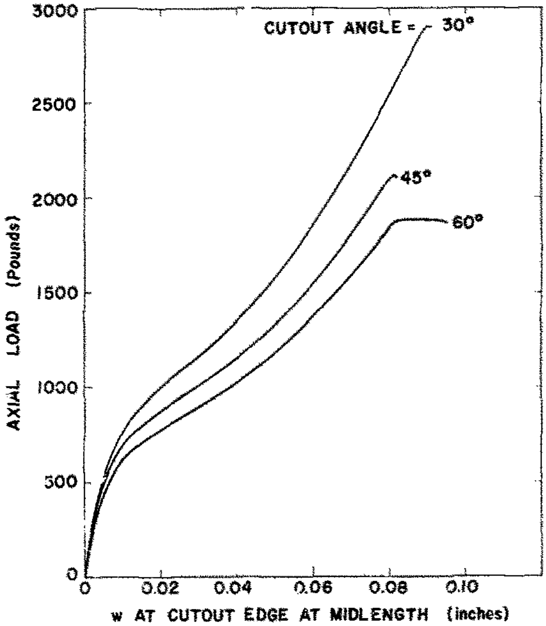


FIG. 3. Load–displacement curves for cylinders with unreinforced cutouts ($t = 0{\cdot}014$).

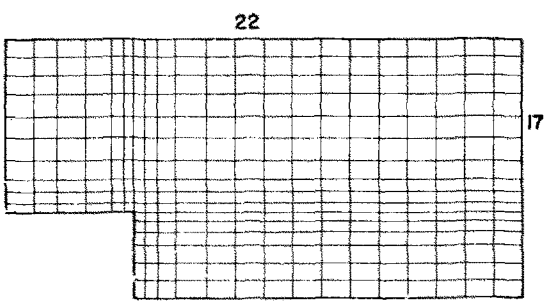


FIG. 4. Finite difference grid for cylinder with 45° cutout.

stiffener was attached like a picture frame around the cutout. The computed critical load as a function of the thickness of the reinforcing frame is shown in Fig. 5. It is clear that this type of reinforcement is very inefficient for this shell. If the reinforcement is light, the cylinder buckles at the midlength of the cutout edge, and at a load only slightly above the load carried by a cylinder with unreinforced cutout. As the thickness of the reinforcing frame is increased the buckle shifts its location to a region above the corner of the cutout and, still, the increase in buckling strength remains slight. This is because the added area causes a stress concentration at the place where the reinforcement ends. The reason that the solid frame could be used to advantage for the cylinder in Ref. [1] appears to be that that cylinder is so much thicker.

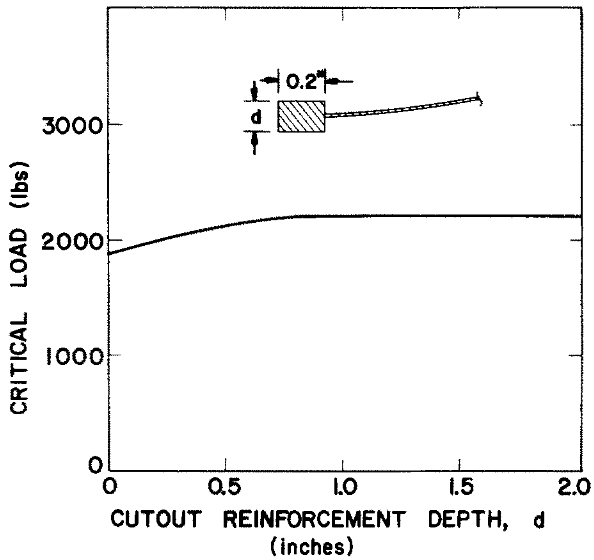


FIG. 5. Effect of solid reinforcement for 60° cutout (t = 0·014).

Clearly the reinforcing stiffener at the cutout edge should have bending stiffness but its area should be as small as possible to reduce stress concentration. A thin angle section stiffener therefore appeared superior to one with the solid rectangular section. Also one might conjecture that for the case of axial compression the stiffening along the curved edges of the cutout may be of little value and that it may be better to sacrifice this part of the frame and instead extend the stiffeners in the axial direction. Linear analysis was used in a preliminary study which established the stiffeners selected as suitable (Figs. 1 and 2). It was also concluded that little would be gained by using a 60° cutout rather than one with a 45° arc and that the latter would be more representative of practical design. The 45° cutout was therefore adopted as the standard for all tests.

Computer results for the collapse load were obtained for three cylinders with 45° cutouts and 0·014 in. thickness. Two were of the type with axial stiffeners only; one with a stiffener thickness of 0·010 in. and one with a thickness of 0·020 in. The third reinforcing configuration had a picture frame reinforcement (Fig. 1) with an angle of 0·020 in. thickness. These reinforcement configurations were then used in the test program. The higher stresses which can be reached in the shell with reinforced cutout makes it necessary to use a finer finite difference grid. The grid selected for analysis of these cylinders had 22 axial and 25 circumferential coordinate lines as shown in Fig. 6. For the cylinders with stringer reinforced cutouts, the maximum displacement shifts away from the cutout edge to a point about 4° of arc from the edge as the load increases. In Fig. 7 an attempt has been made to show how the critical load varies with the thickness of the reinforcement. The data points available are too few to indicate more than the trend. It seems clear, however, that the arrangement with only axial stiffeners is definitely superior.

Of the thinner cylinders ($R/t = 675$) only one was analyzed as the computer time is very high for such shells. The reinforcement chosen for the analysis was type C (see Fig. 2) with an angle stiffener which has an outstanding leg with a reduced height of 0·080 in. For reasonable accuracy in the results, it is necessary to use a very fine grid; the chosen

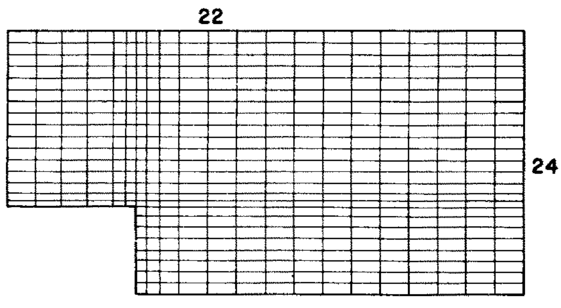


FIG. 6. Finite difference grid for cylinder with 45° reinforced cutout.

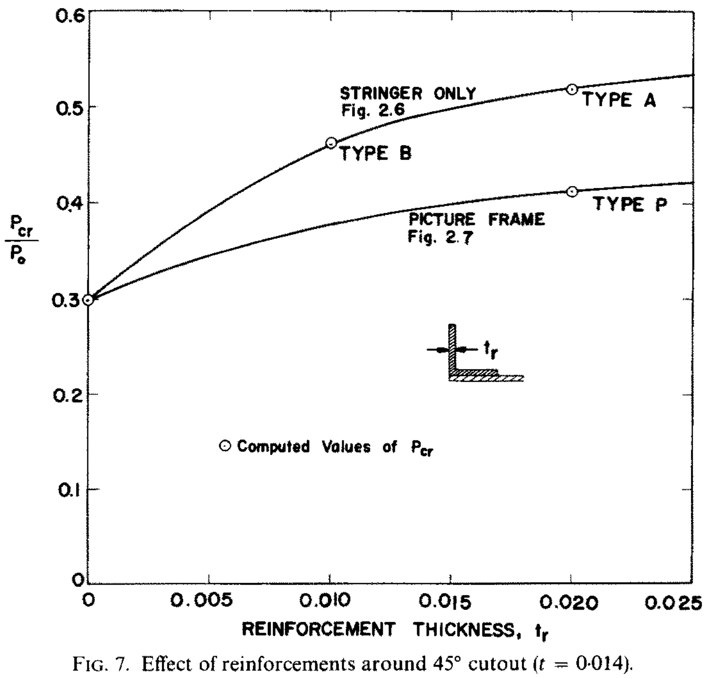


FIG. 7. Effect of reinforcements around 45° cutout ($t = 0{\cdot}014$).

grid with 28 axial and 33 circumferential stations appears to be satisfactory. This stiffener is so weak that the maximum displacement still occurs at the cutout edge.

CORRELATION

The extensive strain measurements for cylinder No. 2 (with thickness 0·014 in. and unreinforced cutouts) offers a good opportunity to compare theoretical and experimental results and thus verify the validity of the computer program. The solid lines in Figs. 8–10 represent computed stresses. The points are the stress values determined by use of the strain gages.

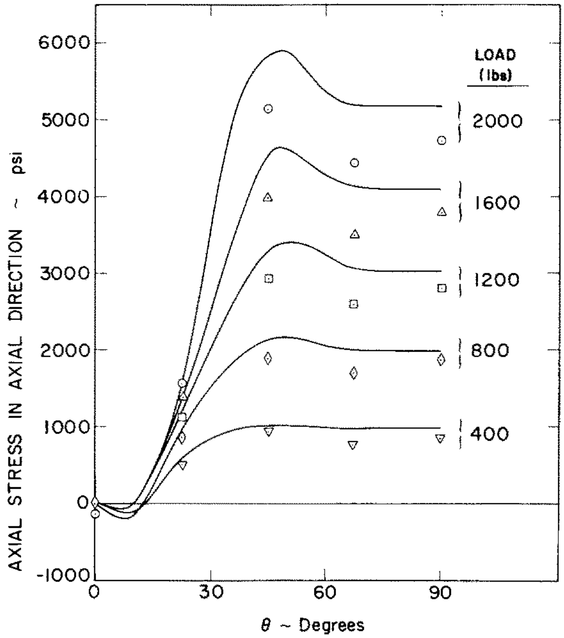


FIG. 8. Axial stress 0·30 in. from end ring (cylinder No. 2).

Figure 8, which shows axial membrane stress 0·30 inches from the end ring, indicates very good agreement between test and theory at all load levels. The agreement deteriorates somewhat as we move away from the cutout. The reason for this appears to be that the theoretical results are for a cylinder with a constant 0·014 in. thickness while the thickness of the actual test cylinder tended to increase to 0·015 or 0·016 in. In Fig. 9, which shows the axial membrane stress at the cylinder midlength, the trend is about the same. At the edge of the cutout the agreement between experimental and theoretical stresses is exceptionally good. Away from the cutout the measured stresses tend to be somewhat lower than computed stresses because the thickness in this area is above nominal.

Bending stresses are generally so small that the dominating influence on these are the small imperfections in the shape of the test cylinder. Only at the edge of the cutout are these stresses big enough to make a comparison between test and theory meaningful. The axial

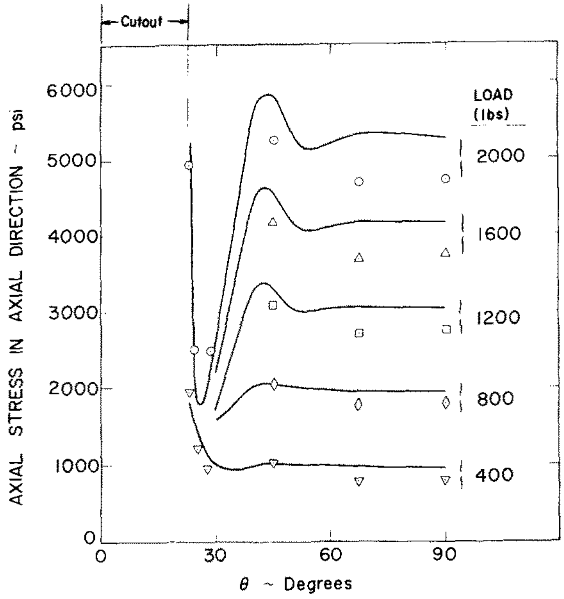


FIG. 9. Axial stress at cylinder No. 2 midheight.

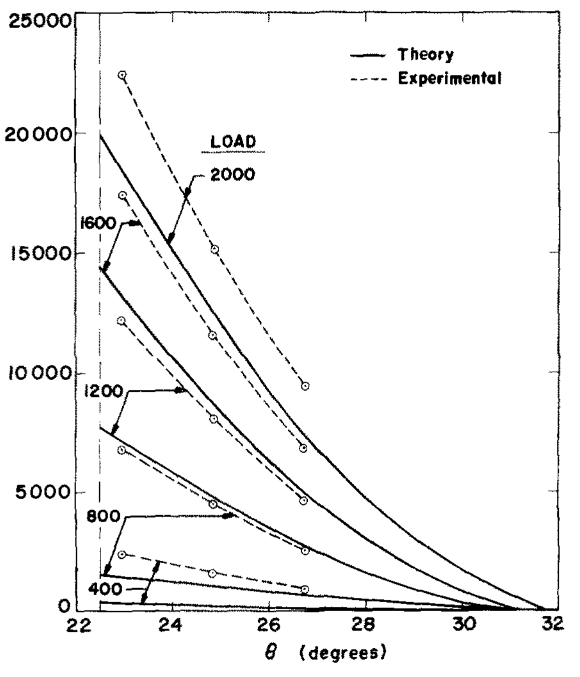


FIG. 10. Bending stress near edge of cutout (cylinder No. 2 midheight).

direction bending stresses at the cylinder midlength and close to the cutout edge are shown in Fig. 10. Here the agreement is seen to be relatively poor for small load levels, at which the influence of imperfections is dominant, but to improve with increasing load.

Figure 11 shows a comparison between theory and measured membrane strains for cylinder No. 4. While these results may be considered typical, more extensive comparisons are made in Ref. [4]. For locations A and C the agreement is generally good. At location B, the agreement is not as good, but it should be pointed out that there is a very steep stress gradient in this region (see Fig. 9), so that the placement of the gage is very critical, or conversely, measurements have a high probability of being "off" because of minor gage misplacement. Taking this into consideration, it is felt that agreement between test and theoretically predicted membrane strains is very good also for the reinforced cylinders.

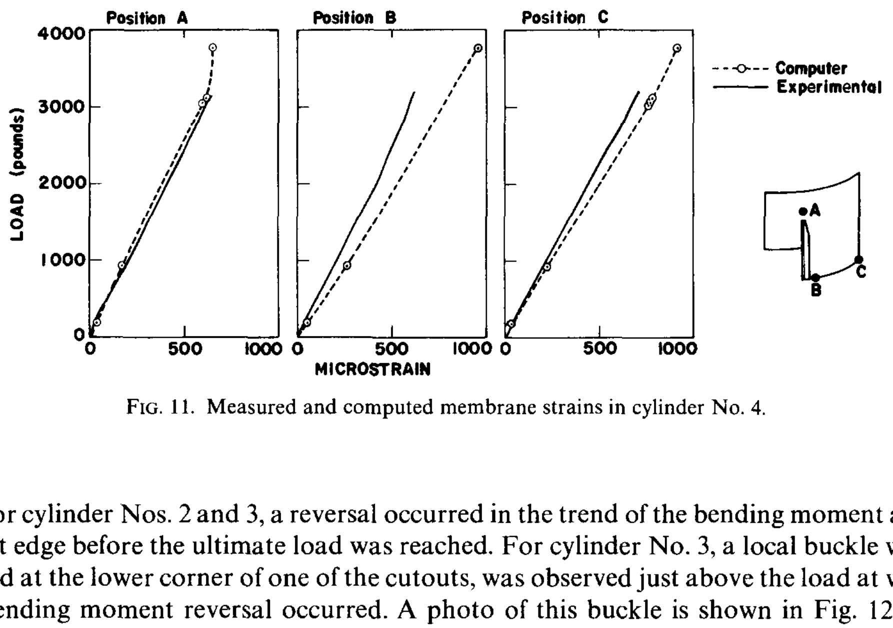


FIG. 11. Measured and computed membrane strains in cylinder No. 4.

For cylinder Nos. 2 and 3, a reversal occurred in the trend of the bending moment at the cutout edge before the ultimate load was reached. For cylinder No. 3, a local buckle which formed at the lower corner of one of the cutouts, was observed just above the load at which the bending moment reversal occurred. A photo of this buckle is shown in Fig. 12. The cylinder carried additional load after the formation of this first buckle and finally collapsed (Fig. 13) upon reaching an axial load of 2170 lb. We feel that the point of the bending stress reversal is the proper load level to compare with the theoretical collapse loads. For cylinder No. 2 the theoretical load is then 2250 lb and the experimental load is 2200 lb. Cylinder No. 3 is somewhat thinner; in the neighborhood of the cutout the thickness was 0·13 in. If the collapse load is assumed to be proportional to the square of the thickness, the thickness corresponding to the test failure load of 2000 lb, is 0·0132 in., which agrees well with the measured thickness. For cylinder No. 1 with a 30° cutout no stress reversal was observed before collapse. The critical load of 2740 lb compares well with the computed load of 2900 lb. (The thickness varies in the neighborhood of a cutout between 0·014 and 0·015 in.) In Fig. 14 the critical load is plotted as a function of the width of the cutout. In addition to the analytical results for 30, 45 and 60° cutouts, we know of course the critical loads foı 0 and 180° cutouts. Due to the limited number of points the curve is rather uncertain, particularly for cutouts between 0 and 30°.

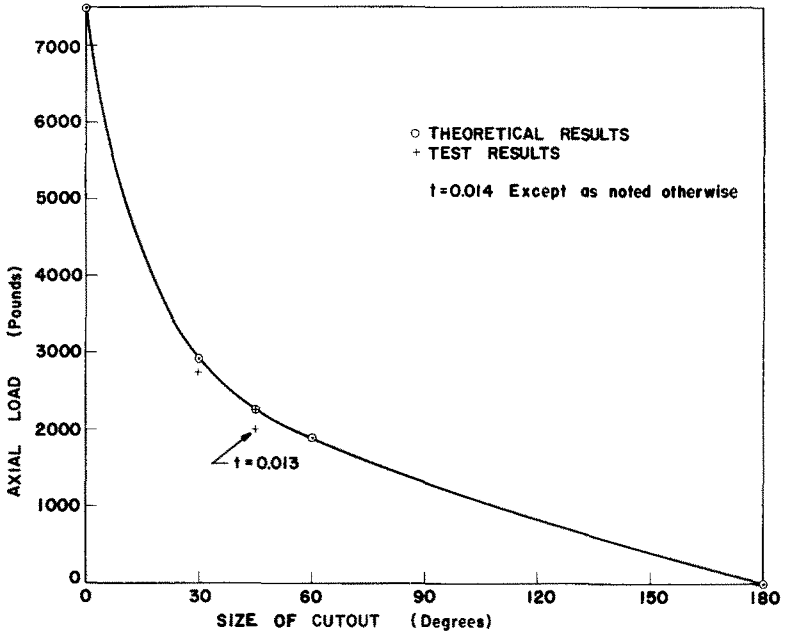


FIG. 14. Critical load vs. cutout angle.

It is seen that in cylinders with reinforced as well as unreinforced cutouts, theory and experiment agree very well on the stress distribution. In addition, for cylinders with unreinforced cutouts, the theory predicts quite accurately the collapse load. In the case of cylinders with reinforced cutouts, it is evident that a reinforced cutout constitutes less of an imperfection than was generally found in these cylinders, so that a knock-down factor based on the imperfection level has to be applied to the computer based nonlinear analysis. This agreement between test and theory is encouraging and is one of the most important conclusions of the program. It indicates that it would be possible to make extensive studies of the efficiency of cutout reinforcement designs primarily on an analytical basis.

It is useful to note that we obtain a reasonably good approximation to the effective axial stress level by dividing the total load by the cross sectional area of the cylinder which remains after the cutout is introduced. One should be cautioned that this remark, as well as the following observations, apply only to the situation in which the load is applied by constant end shortening. This accurately represents the test conditions, and is applicable to many practical problems as well (e.g. collapse of a section of a launch vehicle contained between two large bulkheads). However, cylinders to which a uniform axial edge load is applied will behave quite differently (the interior stress distribution is highly nonuniform and the collapse load will be lower than for the same shell with constant end shortening); such cases have not been studied extensively and are beyond the scope of the present effort.

The maximum stress σ_{cr} which the cylinder can sustain (under constant end shortening), even if the cutout is adequately reinforced, is the critical stress for a complete cylinder. In view of the sensitivity of axially loaded cylinders to geometrical imperfections, a cylinder

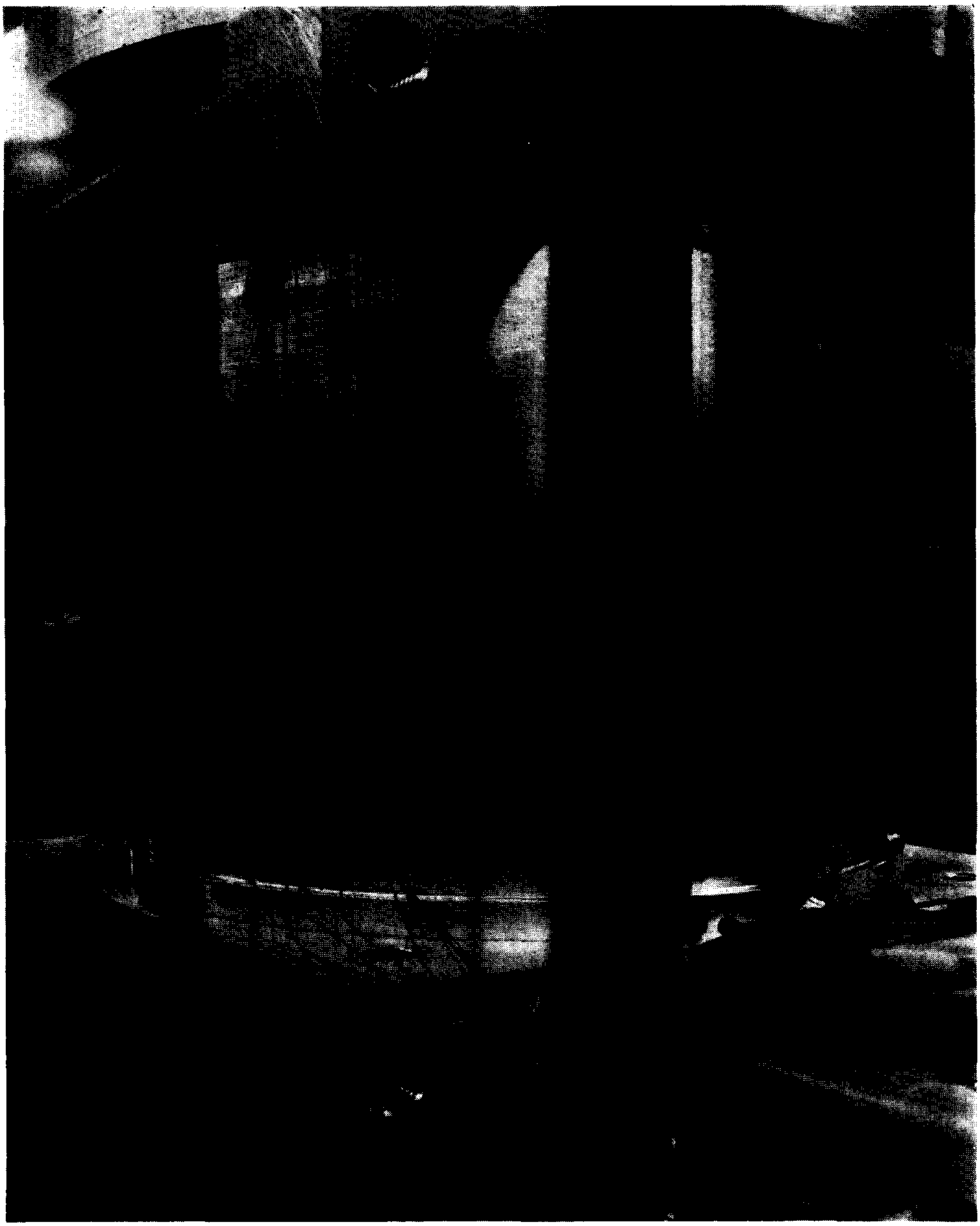


FIG. 12. First buckle at 2050 lb, cylinder No. 3. Cylinder went on to carry 2170 lb.

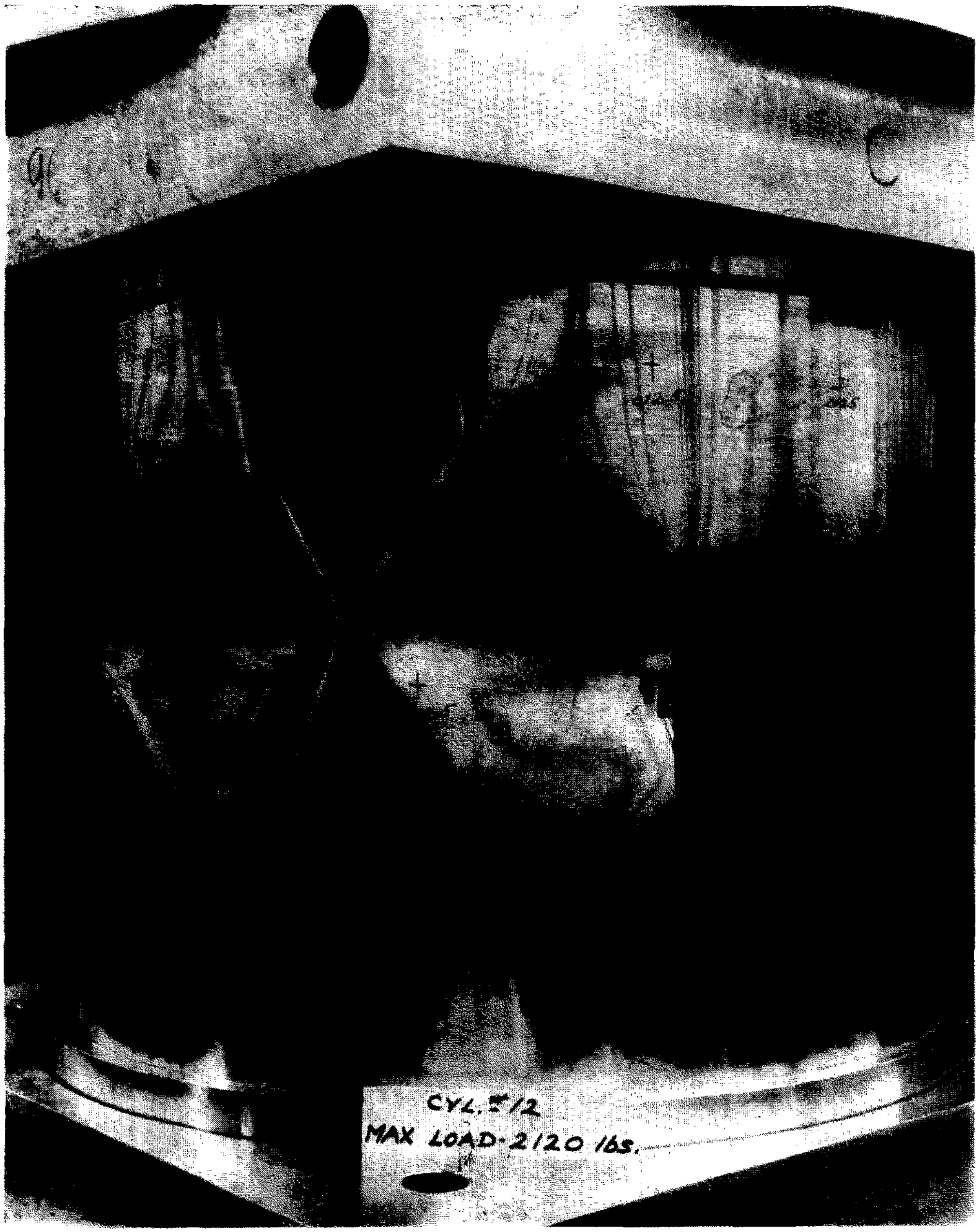


FIG. 13. Cylinder No. 3 after buckling, general view, east side.

without a cutout has a critical axial stress of

$$\sigma_{cr} = \phi\sigma_0 \tag{1}$$

where ϕ is a knock-down factor tied to a probability level depending on the quality of the cylinder and σ_0 is the classical buckling stress for a perfect cylinder without a cutout, i.e.

$$\sigma_0 = 0{\cdot}6Et/R. \tag{2}$$

Thus the maximum load the cylinder can sustain is the critical stress times the net area (assuming two equal unreinforced cutouts 180° apart)

$$P_u = \frac{180-\alpha}{180}\phi P_0 = \psi P_0 \tag{3}$$

where

$$\alpha = \text{angular arc of one cutout};$$

$$P_0 = 2\pi Rt\sigma_0.$$

If the reinforcement around the cutout is inadequate or nonexistent, the shell may collapse at a load significantly less than the upper bound P_u given by equation (3). This collapse load P_{NL} must be determined by a nonlinear analysis. The critical load P_{CR} for the shell is then the smaller of the two loads P_{NL} and P_u.

If the quality parameter ϕ is relatively small the unreinforced cutout may represent a less severe imperfection than those which were present in the complete cylinder. For a given value of ϕ it should be possible then to find the size of a cutout such that the cutout is not the most severe imperfection and therefore reinforcement will not increase the design load for the cylinder. The maximum size of a cutout that can as well be left unreinforced is shown in Fig. 15. The curve is based on computer runs for cylinders with 30, 45

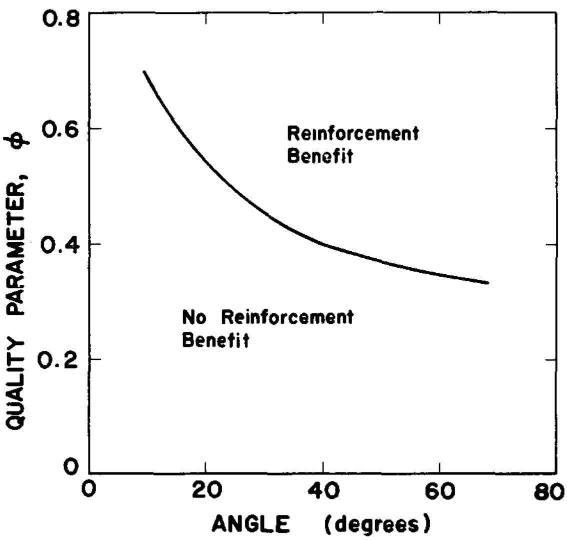


FIG. 15. Reinforcement benefit as a function of cutout arc and cylinder quality parameter.

and 60° cutouts. It is stressed here that the present investigation was not very extensive and the suggestions made here for design procedures must be considered as tentative.

We should notice also that for cylinders with low values of ϕ there is considerable scatter in the test data. Hence if for a cylinder of a given geometry and manufacturing procedure the design load corresponds to $\phi = 0{\cdot}41$ and this value is based on 99 per cent probability, then only 1 per cent of a number of cylinders tested will have a critical load less than $0{\cdot}41\ P_0$. Many cylinders will carry a considerably higher load. If a 30° unreinforced cutout is made in such a cylinder, test results will be concentrated around $P_{CR} = \psi P_0 = 150/180 \times 0{\cdot}41\ P_0 = 0{\cdot}34\ P_0$. Introduction of reinforcement will not change the critical load for the cylinders of poor quality. The lower bound or the 99 per cent limit cannot be much increased through cutout reinforcement, but for the cylinders in the set which are of somewhat better quality the critical load can be raised. Thus the average buckling load would increase with reinforcement for cylinders which are not too far below the curve in Fig. 15.

As the value of ϕ was determined for all test specimens before any cutouts were introduced, it is possible to obtain a preliminary evaluation of the method suggested above [$P_{CR} = \min(P_u, P_{NL})$] by application to all cases for which theoretical as well as experimental results are available. Such an evaluation is made in Table 3. Since it is difficult to take the

TABLE 3. CORRELATION BETWEEN TEST AND THEORY

Specimen No.	ϕ	ψ	P_u	P_{NL}	P_{CR}	P_{EXP}
1	0·545	0·455	3360	2900	2900	2740
2	0·620	0·465	3440	2250	2250	2250†
3	0·578	0·435	3210	2250	2250	2000†
4	0·503	0·375	2780	3700	2780	3190
5	0·538	0·403	2980	3700	2980	2850
6	0·455	0·34	2500	3500	2500	2560
7	0·413	0·31	2290	3100	2290	2600
10	0·45	0·338	1030	1400	1030	1030

Key—ϕ, ψ and P_u, see equations (1) and (3); P_{NL}, theoretical buckling load from nonlinear analysis of perfect shell with cutout; P_{CR}, predicted buckling load (minimum of P_u and P_{TH}); P_{EXP} experimental buckling load.

† Load at which bending strain reversed; this is somewhat lower than total collapse load shown in Table 1.

variable thickness into account and since many of the computer runs were made before the cylinders were manufactured, all calculations here are based on nominal values of the thickness. In view of the thickness variation in any given shell, this approximation is not inappropriate. However, more analysis and additional experiments are needed before this method could be considered an established design procedure. As might be expected, the nonlinear analysis value provides the critical load for all shells with unstiffened cutouts (Nos. 1–3). However, in spite of the very light stiffening used in some case, P_u is critical in all specimens with reinforced cutouts. Any future work should therefore be on cylinders that have even lighter cutout reinforcement and a higher value of the quality parameter ϕ.

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Абстракт—В 1902 году фейлон сделал попытку разрешить проблему упомянутую в заглавии, очевидно, что решение достигнутое тогда до сих пор еще не было уточнено. В настоящей работе предлагается такое уточнение для всех цилиндров с отношением длина-диаметр более 0, 1. Техника собственной функции Литтла и Чайлдза далее развивается в метод в который можно включить особенности напряжений у окружностей конечной плоскости. Наличие этих особенностей препятствует получению надежных результатов другими методами.